Important announcements

- These and future lecture notes will be posted on the course website.
- All recordings will be posted on eClass
- Points of contact:
  1) Piazza
  2) TA office hours
  3) cmput267@ualberta.ca

Math Review

- Sets and Vectors
- functions

Why more rigerous math?
- write complicated stuff conveniently
- divide & conquer
  => allows to do more complex things
- understand papers

Motivation

Supervised Learning:

math review

Learning from a randomly sampled batch of labeled data

# of rooms	price
2	200
4	590
3	350
7	970

Predictor function f
input: # of rooms
output: price

Experience $\xrightarrow{\text{Learner}}$ knowledge

formalize $\Downarrow$

Programs/Algorithms implement functions

Dataset (tuple of tuples) $\xrightarrow{\text{Learner (function)}}$ Predictor = Model = Hypothesis (function)

features	label
$x_1 = (x_{1,1}, x_{1,2})$	y_1
$x_2 = (x_{2,1}, x_{2,2})$	y_2
$\vdots$	$\vdots$
$x_n = (x_{n,1}, x_{n,2})$	y_n

$x_i \in \mathcal{X}$

$y_i \in \mathcal{Y}$

$i = \{1, \dots, n\}$

Ex: $\mathcal{X} = \mathbb{R}^2$

$\mathcal{Y} = \mathbb{R}$

domain   co-domain

$f : \mathcal{X} \to \mathcal{Y}$

f is a function from features to labels

Ex $f(x) = 2x + 1$ $\mathcal{X} = \mathbb{R}$

$A : (\mathcal{X} \times \mathcal{Y})^n \to f : \mathcal{X} \to \mathcal{Y}$

A is a function from dataset to predictors

Ex:

$$f(x) = \begin{cases} y_i & \text{if } x = x_i \text{ for } i \in \{1, \dots, n\} \\ 0 & \text{otherwise} \end{cases}$$

$$\boxed{\begin{array}{l} D = \left((\vec{x}_1, y_1), (\vec{x}_2, y_2), \ldots, (\vec{x}_n, y_n)\right) \\ D \in (\mathcal{X} \times \mathcal{Y})^n \end{array}}$$

contains duplicates

D n feature-label pairs

$\mathcal{X}$ set of features

$\mathcal{Y}$ set of labels/targets

Ex: $\mathcal{X} = \mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$, $\mathcal{Y} = \mathbb{R}$

features	Label
$\vec{x}_1 = (x_{1,1}, x_{1,2})$	y_1
$\vec{x}_2 = (x_{2,1}, x_{2,2})$	y_2
$\vdots$	$\vdots$
$\vec{x}_n = (x_{n,1}, x_{n,2})$	y_n

$\vec{x}_i \in \mathcal{X}$, $y_i \in \mathcal{Y}$, $i \in \{1, \ldots, n\}$

$$\boxed{f : \mathcal{X} \to \mathcal{Y}}$$

f is a function from features to labels

Ex $f(x) = 2x + 1$, $\mathcal{X} = \mathbb{R}$

$$\boxed{\mathcal{A} : (\mathcal{X} \times \mathcal{Y})^n \to \{f \mid f : \mathcal{X} \to \mathcal{Y}\}}$$

$\mathcal{A}$ a function from datasets to predictors

Ex: $\mathcal{A}(D) = f$ where $f : \mathcal{X} \to \mathcal{Y}$

$$f(x) = \begin{cases} y_i & \text{if } x = x_i \text{ for some } i \in \{1, \ldots, n\} \\ 0 & \text{otherwise} \end{cases}$$

Sets

Set: a collection of distinct and unordered objects

Ex: $\{0,1,2\} = \{0,1,2,2\}$, $\{cat, dog\} = \{dog, cat\}$

$\mathbb{N} = \{0,1,2,\dots\}$ natural numbers

$\mathbb{R}$ real numbers, $\emptyset = \{\}$ empty set

Variables as sets: Ω, x, y, z

Ex: $x = \{0,1,2\}$, $\Omega = \{cat, dog\}$

Cardinality: size of the set

Ex: $|x| = 3$, $|\Omega| = 2$, $|\emptyset| = 0$, $|\mathbb{N}|$, $|\mathbb{R}|$ $\quad |\mathbb{N}| \neq |\mathbb{R}|$

Countably Infinite Set: If you can list all the elements in the set

Ex: $|\mathbb{N}|$ = countably infinite, $\mathbb{N} = \{0,1,2,3,\dots\}$

$\{x \in \mathbb{N} \mid x \text{ is even}\} = \{0,2,4,6,\dots\}$

Uncountably Infinite Set: Not countably infinite

Ex: $|\mathbb{R}|$ = uncountably infinite, $\mathbb{R} \neq \{0, 0.0001, 0.0002, \dots\}$

0.00001

$|[0,900]|$ = uncountably infinite

Element of and Subset of sets: $\in, \notin, \subset, \not\subset$

$\Omega = \{cat, dog\}$

$\mathcal{X} = \{0,1,2\}$

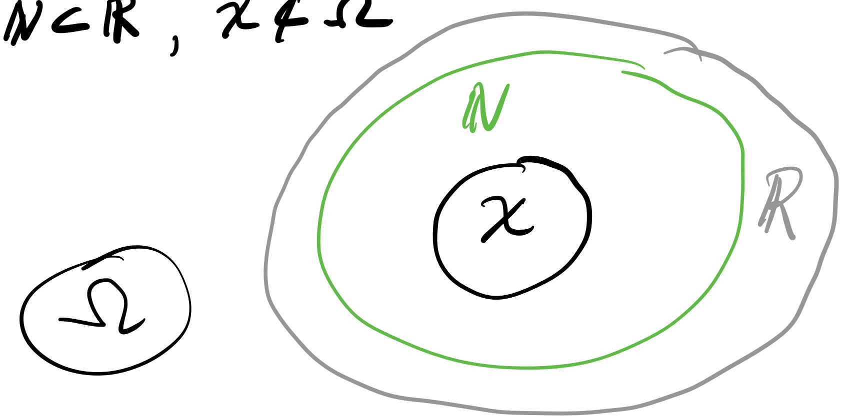
Ex: $cat \in \Omega$ cat is an element of Ω

$cat \notin \mathcal{X}$

$0 \in \mathbb{R}, 1 \in \mathbb{R}, -2 \in \mathbb{R}, \frac{1}{2} \in \mathbb{R}, 0.23 \in \mathbb{R}, \pi \in \mathbb{R}, \infty \notin \mathbb{R}$

$\mathcal{X} \subset \mathbb{N}, \mathcal{X} \subset \mathbb{R}, \mathbb{N} \subset \mathbb{R}, \mathcal{X} \not\subset \Omega$

$\mathcal{X} \subset \mathcal{X}, \Omega \subset \Omega$



Intervals: Continuous subset of $\mathbb{R}$

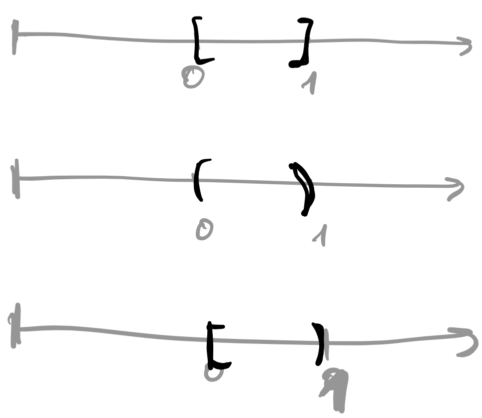
Closed: $[0,1] \subset \mathbb{R}$

$[0,1] \not\subset \mathbb{N}$

open: $(0,1) \subset \mathbb{R}$, $0 \notin (0,1)$

half-open: $[0,1) \subset \mathbb{R}$, $1 \notin [0,1)$

$[0,\infty)$

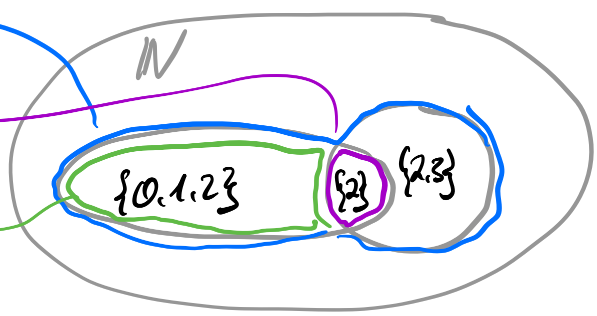


Unions, Intersections, Set Difference: $\cup, \cap, \setminus$

$\{0,1,2\} \cup \{2,3\} = \{0,1,2,3\}$

$\{0,1,2\} \cap \{2,3\} = \{2\}$

$\{0,1,2\} \setminus \{2,3\} = \{0,1\}$



Complement: $\mathcal{X}^c$ -> define universal set

Ex: $\mathcal{U} = \mathbb{N}$, $\mathcal{X}^c = \{0,1,2\}^c = \mathcal{U} \setminus \mathcal{X} = \{3,4,5,\dots\}$

Set Builder Notation: $\{\text{element} \mid \text{property}\}$ "such that"

Ex: $\{x \in \mathbb{N} \mid x \leq 2\} = \{0,1,2\}$

$\{x \in \mathbb{N} \mid x \text{ is even}\} = \{0,2,4,\dots\}$

$\{x \in \mathbb{N} \mid x \text{ is prime}\} = \{2,3,5,7,11,\dots\}$

$\{x \in \mathbb{N} \mid x \in \{0,1,2\} \text{ and } x \notin \{2,3\}\} = \{0,1,2\} \setminus \{2,3\}$

"define sets in a convenient way"

Power set: $P(\mathcal{Z}) = \{S \mid S \subset \mathcal{Z}\}$ the set of all subsets of $\mathcal{Z}$

Ex: $P(\{0,1\}) = \{\emptyset, \{0\}, \{1\}, \{0,1\}\}$

Tuple: a collection of ordered objects with duplicates allowed

Ex: $(0,1,2) \neq (0,1,2,2)$, $(\text{cat}, \text{dog}) \neq (\text{dog}, \text{cat})$ $\boxed{\text{This is not an interval!}}$

only variable and element of properties apply

$\Omega' = (0,1,2)$, $\mathcal{X}' = (\text{cat}, \text{dog})$, $2 \in \Omega'$, $2 \notin \mathcal{X}'$

Invalid: $\Omega' \subset \mathcal{X}'$, $\Omega' \cup \mathcal{X}'$, $\Omega' \cap \mathcal{X}'$, ...

Cartesian products (creating tuples): set × set

Ex: $\Omega \times \mathcal{X} = \{\text{cat}, \text{dog}\} \times \{0,1,2\}$

$= \{(\text{cat},0), (\text{cat},1), (\text{cat},2), (\text{dog},0), (\text{dog},1), (\text{dog},2)\}$

$= \{(a,b) \mid a \in \Omega, b \in \mathcal{Z}\}$

Ex: $\mathbb{R}^2 = \mathbb{R}\times\mathbb{R} = \{(a,b) \mid a\in\mathbb{R},\ b\in\mathbb{R}\}$

$(0,2)\in\mathbb{R}^2$, $(-\frac{1}{10},\pi)\in\mathbb{R}^2$

Ex: $[0,1]^2 = [0,1]\times[0,1] = \{(a,b)\mid a\in[0,1],\ b\in[0,1]\}$

Ex: $\mathbb{R}^3 = \{(a,b,c)\mid a\in\mathbb{R},\ b\in\mathbb{R},\ c\in\mathbb{R}\}$

Ex: $\mathcal{X}=\mathbb{R}^2$, $\mathcal{Y}=\mathbb{R}$

$\mathcal{X}\times\mathcal{Y} = \{(x,y)\mid x\in\mathcal{X},\ y\in\mathcal{Y}\}$ our dataset from before

$\mathbb{R}^3 \neq \mathbb{R}^2 = \mathcal{X}$

$((2,2),200)\in\mathcal{X}\times\mathcal{Y}$, $(\overbrace{(2,12,1)},200)\notin\mathcal{X}\times\mathcal{Y}$

$\mathcal{X}\times\mathcal{Y} = (\mathbb{R}^2)\times\mathbb{R} \neq \mathbb{R}^2\times\mathbb{R} = \mathbb{R}^3$, $(2,2,200)\in\mathbb{R}^3$

order is important

Ex: $(\mathcal{X}\times\mathcal{Y})^n = \underbrace{(\mathcal{X}\times\mathcal{Y})}_{1}\times\underbrace{(\mathcal{X}\times\mathcal{Y})}_{2}\times\underbrace{\cdots}_{\cdots}\times\underbrace{(\mathcal{X}\times\mathcal{Y})}_{n}$ Duplicates are allowed

$D = (((2,2),200),\ ((4,10),450),\ \dots,\ ((2,2),200))$