Exercise 1: What do the elements of the set $\mathbb{R}^3$ look like? ☐

Exercise 2: What do the elements of the set $(\mathbb{R}^2) \times \{0,1\}$ look like? ☐

Exercise 3: If $\mathcal{X} \subseteq \mathbb{R}^d$ and $\mathcal{Y} \subseteq \mathbb{R}$, then is $(\mathcal{X} \times \mathcal{Y})^n$ a set or a tuple? ☐

Exercise 4: If $\mathcal{X} \subseteq \mathbb{R}^d$ and $\mathcal{Y} \subseteq \mathbb{R}$, then what does an element of $\mathcal{X} \times \mathcal{Y}$ look like? ☐

Exercise 5: If $\mathcal{X} \subseteq \mathbb{R}^d$ and $\mathcal{Y} \subseteq \mathbb{R}$, then what does an element of $(\mathcal{X} \times \mathcal{Y})^n$ look like? ☐

Ex 1: $\mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(a,b,c) \mid a \in \mathbb{R}, b \in \mathbb{R}, c \in \mathbb{R}\}$

$$= \{(a,b,c) \mid a,b,c \in \mathbb{R}\}$$

$$(1,2,3) \in \mathbb{R}^3, \quad (\pi, -3, \tfrac{1}{17}) \in \mathbb{R}^3, \quad (1,2) \notin \mathbb{R}^3$$

Ex 2: $(\mathbb{R}^2) \times \{0,1\} = \{(a,b) \mid a \in \mathbb{R}, b \in \mathbb{R}\} \times \{0,1\}$

$$= \{((a,b),c) \mid a \in \mathbb{R}, b \in \mathbb{R}, c \in \{0,1\}\}$$

$$\neq \{((a,b),\{0,1\}) \mid a \in \mathbb{R}, b \in \mathbb{R}\}$$

$$((1,2),0), \; ((0,2.7),1), \; ((0,2.7),0) \in (\mathbb{R}^2) \times \{0,1\}$$

Ex 3: $(\mathcal{X} \times \mathcal{Y})^n = (\mathcal{X} \times \mathcal{Y}) \times (\mathcal{X} \times \mathcal{Y}) \times \dots \times (\mathcal{X} \times \mathcal{Y})$ $\quad i \in \mathbb{N}, i > 0$

$i \in \{x \in \mathbb{N} \mid x > 0\}$

$$= \{\underbrace{(z_1, z_2, \dots, z_n)}_{\text{a dataset}} \mid z_i \in \underbrace{\mathcal{X} \times \mathcal{Y}}, \; i \in \{1, \dots, n\}\}$$

Set of all Datasets

D

$\mathcal{X} = \mathbb{R}^d$, $\mathcal{Y} = \mathbb{R}$

$$\in \underbrace{((x_1, x_2, \dots, x_d), y)}_{\text{a feature-label pair}}$$

$f : \mathbb{R} \to \mathbb{R}$ $\qquad$ $\mathcal{A} : (\mathcal{X} \times \mathcal{Y})^n \to$

$f(x) = x^2$ $\qquad$ $\mathcal{A}(D) = f$

Exercise 4: If $\mathcal{X} \in \mathbb{R}^d$ and $\mathcal{Y} \in \mathbb{R}$, then what does an element of $\mathcal{X} \times \mathcal{Y}$ look like? $\square$

Exercise 5: If $\mathcal{X} \in \mathbb{R}^d$ and $\mathcal{Y} \in \mathbb{R}$, then what does an element of $(\mathcal{X} \times \mathcal{Y})^n$ look like? $\square$

Ex 4: $\mathcal{X} = \mathbb{R}^d, \mathcal{Y} = \mathbb{R}$

$$\mathcal{X} \times \mathcal{Y} = (\mathbb{R}^d) \times \mathbb{R}$$
$$= \{(x, y) \mid x \in \mathbb{R}^d, y \in \mathbb{R}\}$$
$$= \{((x_1, x_2, \ldots, x_d), y) \mid x_1, x_2, \ldots, x_d, y \in \mathbb{R}\}$$

$((x_1, \ldots, x_d), y) \in \mathcal{X} \times \mathcal{Y} \quad x_1, \ldots, x_d, y \in \mathbb{R}$

$((1, \ldots, 2), 2) \in$

Exercise 6: Suppose you wanted to keep information of house being sold. You decide to use two features to represent each house and to keep track of the price (an element of $[0, \infty)$) it was sold at. The first feature was the number of rooms (an natural number), the second feature was the square footage (an element of $[0, \infty)$). How would you write the set of all possible houses that are represented in this way? Elements of this set should look like $((x_1, x_2), y)$ where $x_1 \in \mathbb{N}, x_2 \in \mathbb{R}$ and $y \in \mathbb{R}$. $\square$

Exercise 7: How would you write the set that contains all the tuples of the form $\underbrace{(((x_{1,1}, x_{2,1}), y_1), \ldots, ((x_{n,1}, x_{n,2}), y_n))}_{=D_1}$ where $x_{i,1}, x_{i,2} \in \mathbb{R}$ and $y_i \in \{0, 1\}$ for all $i \in \{1, \ldots, n\}$. $\square$

$(\mathcal{X} \times \mathcal{Y})^n$, where $\mathcal{X} = \mathbb{R}^2, \mathcal{Y} = \{0, 1\}$

$= ((\mathbb{R}^2) \times \mathbb{R})^n$

$\mathcal{X} \in \mathbb{R}^2 \quad \mathcal{X} = (1, 2)$

$\mathbb{R}^2 = \{(a, b) \mid a \in \mathbb{R}, b \in \mathbb{R}\}$

$((\mathbb{R}^2) \times \mathbb{R})^n = ((\mathcal{X}) \times \mathcal{Y})^n$ where $\mathcal{X} = \mathbb{R}^2$, $\mathcal{Y} = \mathbb{R}$

$D_1 \in \{D_1, D_2\}$

$4 \in \{4, 5\}$

$f: \mathbb{R} \to \mathbb{R}$

$\mathcal{A}: \{D_1, D_2\} \to \{f \ldots\}$

$$z^3 = z \times z \times z$$

$$z^2 = (x+y)^2$$

$$z = x \times y \qquad (x \times y)^3$$

$$z = x + y$$