Homework Assignment 2

Due: Friday, September 27, 2024, 11:59 p.m. Mountain time

Total marks: 20

Policies:

For all multiple-choice questions, note that multiple correct answers may exist. However, selecting an incorrect option will cancel out a correct one. For example, if you select two answers, one correct and one incorrect, you will receive zero points for that question. Similarly, if the number of incorrect answers selected exceeds the correct ones, your score for that question will be zero. Please note that it is not possible to receive negative marks. **You must select all the correct options to get full marks for the question.**

While the syllabus initially indicated the need to submit a paragraph explaining the use of AI or other resources in your assignments, this requirement no longer applies as we are now utilizing eClass quizzes instead of handwritten submissions. Therefore, you are **not** required to submit any explanation regarding the tools or resources (such as online tools or AI) used in completing this quiz.

This PDF version of the questions has been provided for your convenience should you wish to print them and work offline.

Only answers submitted through the eClass quiz system will be graded. Please do not submit a written copy of your responses.

Question 1. [1 MARK]

Suppose you flip three coins. Suppose the first coin is represented by random variable $X_1 \in \{0,1\}$, the second coin by $X_2 \in \{0,1\}$, and the third coin by $X_3 \in \{0,1\}$. Which of the following is the outcome space of the random variable $X = (X_1, X_2, X_3)$?

a. $\{0,1\}^3$

b. $\{1,2,3\}$

c. $\{(x_1, x_2, x_3) \mid x_1 \in \{0,1\}, x_2 \in \{0,1\}, x_3 \in \{0,1\}\}$

d. $\{0,1\}$

Question 2. [1 MARK]

Suppose you roll a fair twenty-sided die. The outcome space is $\mathcal{X} = \{1, 2, 3, \ldots, 20\}$. Which of the following is an event?

a. $\{0,1,2\}$

b. $\{x \in \mathcal{X} \mid x > 10\}$

c. 12

d. $\{12\}$

Question 3. [1 MARK]

Which of the following is an event from the outcome space $\mathcal{X} \times \mathcal{Y}$, where $\mathcal{X} = \mathbb{R}^2$ and $\mathcal{Y} = \mathbb{R}$?

a. $\mathcal{X} \times \mathcal{Y}$

b. A function $f : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}$

c. $((1, 2), 3)$

d. $\{((x_1, x_2), y) \in \mathcal{X} \times \mathcal{Y} \mid y \geq 300\}$

Question 4. [1 MARK]

Which of the following is an event from the outcome space $(\mathcal{X} \times \mathcal{Y})^n$, where $\mathcal{X} = \mathbb{R}^2$ and $\mathcal{Y} = \mathbb{R}$?

a. $(\mathcal{X} \times \mathcal{Y})^n$

b. $(((x_{1,1}, x_{1,2}), y_1), \ldots, ((x_{n,1}, x_{n,2}), y_n))$ where $x_{i,1}, x_{i,2}, y_i \in \mathbb{R}$ for all $i \in \{1, \ldots, n\}$

c. $\{(((x_{1,1}, x_{1,2}), y_1), \ldots, ((x_{n,1}, x_{n,2}), y_n)) \in (\mathcal{X} \times \mathcal{Y})^n \mid y_i \geq 300 \text{ for all } i \in \{1, \ldots, n\}\}$

d. $\{(((x_{1,1}, x_{1,2}), y_1), \ldots, ((x_{n,1}, x_{n,2}), y_n)) \in (\mathcal{X} \times \mathcal{Y})^n \mid x_{i,1} \geq 3 \text{ and } y_i \geq 300 \text{ for all } i \in \{1, \ldots, n\}\}$

Question 5. [1 MARK]

Suppose you have a random variable Y representing the house prices in a city. You know Y is distributed according to the normal distribution with mean $100,000$ and standard deviation $10,000$. Is the following True or False? Y is a continuous random variable.

Question 6. [1 MARK]

Suppose you have a random variable X representing the age of houses in a city. You know X is distributed according to the continuous uniform distribution with outcome space $\mathcal{X} = [10, 20]$. Let p be the pdf of X. Is the following True or False? $p(17)$ is the probability that $X = 17$.

Question 7. [1 MARK]

For a continuous random variable $X \in \mathcal{X}$, we know that $\mathbb{P}(X = x) = 0$ for all $x \in \mathcal{X}$. How is it possible that the pdf $p(x) \neq 0$ for all $x \in \mathcal{X}$?

a. Because $p(x)$ is not the probability of x. Instead, it is just a function that we can integrate over to get a probability.

b. Because the pdf measures the probability mass at each point.

c. Because the pdf is always zero for continuous variables.

d. Because the random variable is discrete.

Question 8. [1 MARK]

Suppose that $Y \in \mathbb{R}$ is distributed according to Laplace(10, 2). Is the following True or False? The probability distribution of Y is $\mathbb{P}(Y = y) = \frac{1}{4} \exp\left(-\frac{|y-10|}{2}\right)$ for $y \in \mathbb{R}$.

Question 9. [1 MARK]

Suppose you have two discrete random variables $X \in \{0,1\}$ and $Y \in \{1,2,3\}$. The joint probability mass function (pmf) of X and Y is given by the following values:

$$p(0,1) = \frac{1}{24}, \quad p(0,2) = \frac{1}{12}, \quad p(0,3) = \frac{1}{3},$$

$$p(1,1) = \frac{1}{6}, \quad p(1,2) = \frac{3}{24}, \quad p(1,3) = \frac{3}{12}.$$

Which of the following is the marginal pmf of X?

a. $p_X(0) = \frac{11}{24}, \quad p_X(1) = \frac{13}{24}$

b. $p_X(0) = \frac{1}{2}, \quad p_X(1) = \frac{1}{2}$

c. $p_X(0) = \frac{7}{12}, \quad p_X(1) = \frac{5}{12}$

d. $p_X(0) = \frac{5}{12}, \quad p_X(1) = \frac{7}{12}$

Question 10. [1 MARK]

Using the same random variables X and Y from the previous question, which of the following is the conditional pmf $p_{Y|X}(y|x)$?

a.

$$p_{Y|X}(1|0) = \frac{1}{11}, \quad p_{Y|X}(2|0) = \frac{2}{11}, \quad p_{Y|X}(3|0) = \frac{8}{11}$$

$$p_{Y|X}(1|1) = \frac{4}{13}, \quad p_{Y|X}(2|1) = \frac{3}{13}, \quad p_{Y|X}(3|1) = \frac{6}{13}$$

b.

$$p_{Y|X}(1|0) = \frac{1}{3}, \quad p_{Y|X}(2|0) = \frac{1}{3}, \quad p_{Y|X}(3|0) = \frac{1}{3}$$

$$p_{Y|X}(1|1) = \frac{1}{3}, \quad p_{Y|X}(2|1) = \frac{1}{3}, \quad p_{Y|X}(3|1) = \frac{1}{3}$$

c.

$$p_{Y|X}(1|0) = \frac{1}{2}, \quad p_{Y|X}(2|0) = \frac{1}{4}, \quad p_{Y|X}(3|0) = \frac{1}{4}$$

$$p_{Y|X}(1|1) = \frac{1}{4}, \quad p_{Y|X}(2|1) = \frac{1}{4}, \quad p_{Y|X}(3|1) = \frac{1}{2}$$

d.

$$p_{Y|X}(1|0) = \frac{1}{6}, \quad p_{Y|X}(2|0) = \frac{1}{3}, \quad p_{Y|X}(3|0) = \frac{1}{2}$$

$$p_{Y|X}(1|1) = \frac{2}{7}, \quad p_{Y|X}(2|1) = \frac{1}{7}, \quad p_{Y|X}(3|1) = \frac{4}{7}$$

Question 11. [1 MARK]

Based on the previous two questions, determine whether the following statement is True or False: The random variables X and Y are independent.

Question 12. [1 MARK]

Let $X = (X_1, \ldots, X_n) \in \{0, 1\}^n$ be a random variable representing the outcome of n coin flips. Let $\mathbb{P}_X$ be the distribution of X. Let $\mathbb{P}_{X_i}$ be the marginal distribution of X_i for each $i \in \{1, \ldots, n\}$. Assume $X_1, \ldots, X_n$ are independent and identically distributed with $\mathbb{P} = \text{Bernoulli}(0.7)$, meaning each flip results in heads (1) with probability 0.7 and tails (0) with probability 0.3. Let

$$\mathcal{E} = \{(x_1, \ldots, x_n) \mid x_n = 1 \text{ and } x_i = 0 \text{ for all } i \in \{1, \ldots, n-1\}\}$$

be the event that you get tails for the first $n-1$ flips and then heads on flip n. What is $\mathbb{P}_X(\mathcal{E})$ if $n = 4$?
(Do not write your answer as a fraction. Instead, express it as a decimal number, rounded to two decimal places if necessary. For example, write 0.33 instead of 1/3.)

Question 13. [1 MARK]

Let $X \in \mathcal{X}$ be a random variable. Let $g : \mathcal{X} \to \mathcal{F}$, where

$$\mathcal{F} = \{f \mid f : \mathcal{X} \to \mathbb{R} \text{ and } f(x) = xw \text{ where } w \in \mathbb{R}\}.$$

Is $g(X)$ a random variable and what is its outcome space?

a. Yes, $\mathcal{F}$

b. Yes, $\mathbb{R}$

c. No, $\mathcal{F}$

d. No, $\mathbb{R}$

Question 14. [1 MARK]

If $X_1, \ldots, X_n$ are random variables with the same distribution $\mathcal{N}(\mu, \sigma^2)$, what is the expected value of $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$?

a. μ

b. σ^2

c. $\frac{\mu}{n}$

d. $n\mu$

Question 15. [1 MARK]

Let X be a random variable with outcome space $\mathcal{X} = \{a, b, c\}$ and pmf $p(a) = 0.1, \quad p(b) = 0.2, \quad p(c) = 0.7$. The function $f(x)$ is given by:

$$f(a) = 10, \quad f(b) = 5, \quad f(c) = \frac{10}{7}.$$

What is $\mathbb{E}[f(X)]$?
(Do not write your answer as a fraction. Instead, express it as a decimal number, rounded to two decimal places if necessary. For example, write 0.33 instead of 1/3.)

Question 16. [1 MARK]

Alberta Hospital occasionally has electrical problems. It can take some time to find the problem, though it is always found in no more than 10 hours. The amount of time is variable; for example, one time it might take 0.3 hours, and another time it might take 5.7 hours. The time (in hours) necessary to find and fix an electrical problem at Alberta Hospital is a random variable X, whose density is given by the following uniform distribution:

$$p(x) = \frac{1}{10} \quad \text{if } 0 \leq x \leq 10,$$

$$p(x) = 0 \quad \text{otherwise.}$$

Such electrical problems can be costly for the Hospital, more so the longer it takes to fix it. The cost of an electrical breakdown of duration x is $C(x) = x^3$. What is the expected cost of an electrical breakdown?
(Do not write your answer as a fraction. Instead, express it as a decimal number, rounded to two decimal places if necessary. For example, write 0.33 instead of 1/3.)

Question 17. [1 MARK]

Let N be a random variable that takes values based on the roll of a biased four-sided die. The pmf of N is:

$$p_N(n) = \frac{n}{10} \quad \text{for } n \in \{1, 2, 3, 4\},$$

$$p_N(n) = 0 \quad \text{otherwise.}$$

What is $\mathbb{E}[N]$?
(Do not write your answer as a fraction. Instead, express it as a decimal number, rounded to two decimal places if necessary. For example, write 0.33 instead of 1/3.)

Question 18. [1 MARK]

Let everything be as defined in the previous question. What is $\text{Var}[N]$?
(Do not write your answer as a fraction. Instead, express it as a decimal number, rounded to two decimal places if necessary. For example, write 0.33 instead of 1/3.)

Question 19. [1 MARK]

Let everything be as defined in the previous two questions. Given the random variable N, a biased coin is flipped with the random variable X representing the outcome of the coin flip. If $X = 1$ then the coin shows heads and if $X = 0$ the coin shows tails. Assume the conditional pmf of N given $X = 1$ is:

$$p_{N|X}(1|1) = \frac{1}{7}, \quad p_{N|X}(2|1) = \frac{3}{14}, \quad p_{N|X}(3|1) = \frac{2}{7}, \quad p_{N|X}(4|1) = \frac{5}{14}.$$

What is the conditional expectation $\mathbb{E}[N|X = 1]$ (i.e., the expectation with respect to this conditional pmf)?
(Do not write your answer as a fraction. Instead, express it as a decimal number, rounded to two decimal places if necessary. For example, write 0.33 instead of 1/3.)

Question 20. [1 MARK]

Let $X_1, \ldots, X_n$ be independent and identically distributed random variables with distribution $\mathcal{N}(\mu, \sigma^2)$. What is $\text{Var}\left(\sum_{i=1}^n X_i\right)$ in terms of μ and σ^2?

a. σ^2

b. $n\sigma^2$

c. $\frac{\sigma^2}{n}$

d. $n^2\sigma^2$