Homework Assignment 1

Due: Friday, September 20, 2024, 11:59 p.m. Mountain time
Total marks: 25

Policies:

For all multiple-choice questions, note that multiple correct answers may exist. However, selecting an incorrect option will cancel out a correct one. For example, if you select two answers, one correct and one incorrect, you will receive zero points for that question. Similarly, if the number of incorrect answers selected exceeds the correct ones, your score for that question will be zero. Please note that it is not possible to receive negative marks.

While the syllabus initially indicated the need to submit a paragraph explaining the use of AI or other resources in your assignments, this requirement no longer applies as we are now utilizing eClass quizzes instead of handwritten submissions. Therefore, you are **not** required to submit any explanation regarding the tools or resources (such as online tools or AI) used in completing this quiz.

This PDF version of the questions has been provided for your convenience should you wish to print them and work offline.

Only answers submitted through the eClass quiz system will be graded. Please do not submit a written copy of your responses.

Question 1. [1 MARK]

Is the following True or False. $(\mathbb{R}^2) \times \mathbb{R}$ is a set.

Question 2. [1 MARK]

Is the following True or False. $(((1,2),3),((1/2,1/3),1/4))$ is a tuple.

Question 3. [1 MARK]

Is the following True or False. A tuple can have duplicates.

Question 4. [1 MARK]

Which of the following is an element of $\mathbb{N}^4$?

a. $(1,2,3,4)$

b. $(1,2,3)$

c. $(1,2,3,4,5)$

d. $(1,2,3,\pi)$

Question 5. [1 MARK]

Which of the following is an element of $\mathcal{X} \times \mathcal{Y}$ where $\mathcal{X} = \mathbb{R}^3$ and $\mathcal{Y} = \mathbb{R}$?

a. $(1,2,3)$

b. $((1,2,3),(1,2,3))$

c. $((1, 2, 3), 4)$

d. $(1, (2, 3, 4))$

Question 6. [1 MARK]

Let $\mathcal{D} = ((\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_n, y_n))$ where $\mathbf{x}_i \in \mathbb{R}^d$ and $y_i \in \{0, 1\}$ for all $i \in \{1, \ldots, n\}$. Then, how would you write the set of all possible $\mathcal{D}$?

a. $((\mathbb{R}^d) \times \{0, 1\})^n$

b. $(\mathcal{X} \times \mathcal{Y})^n$ where $\mathcal{X} = \mathbb{R}^d$ and $\mathcal{Y} = \{0, 1\}$

c. $\{(z_1, \ldots, z_n) \mid z_i \in \mathbb{R}^d \times \mathbb{R},\ i \in \{1, \ldots, n\}\}$

d. $\{((\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_n, y_n)) \mid \mathbf{x}_i \in \mathbb{R},\ y_i \in \{0, 1\},\ i \in \{1, \ldots, n\}\}$

Question 7. [1 MARK]

Suppose you wanted to keep information of houses being sold. You decide to use two features to represent each house and to keep track of the price (an element of $[0, \infty)$) it was sold at. The first feature was the number of rooms (a natural number), the second feature was age (an element of $[0, \infty)$). How would you write the set of all possible houses that are represented in this way? Elements of this set should look like $((x_1, x_2), y)$ where x_1 represents the number of rooms, x_2 represents the age, and y represents the price.

a. $(\mathbb{N} \times [0, \infty)) \times [0, \infty)$

b. $\mathbb{N} \times [0, \infty) \times [0, \infty)$

c. $(\mathbb{N} \times [0, \infty)) \times \mathbb{N}$

d. $\mathbb{R}^2 \times [0, \infty)$

Question 8. [1 MARK]

Let $f : (\mathbb{R}^3) \times (\mathbb{R}^3) \to \mathcal{Y}$ be such that $f(\mathbf{x}, \mathbf{w}) = \mathbf{x}^\top \mathbf{w}$, where $\mathbf{x}, \mathbf{w}$ are vectors. If $\mathbf{x} = (1, 4, 2)^\top$ and $\mathbf{w} = (1, 2, 3)^\top$, then what is $f(\mathbf{x}, \mathbf{w})$?

Question 9. [1 MARK]

Let $f : \mathbb{R} \to \mathcal{Y}$ be such that $f(x) = 4 + x^2$. What is the range of f, and what is a valid codomain $\mathcal{Y}$?

a. The range of f is $[4, \infty)$, and a valid codomain is $\mathbb{R}$.

b. The range of f is $[0, \infty)$, and a valid codomain is $[0, \infty)$.

c. The range of f is $[4, \infty)$, and a valid codomain is $[4, \infty)$.

d. The range of f is $\mathbb{R}$, and a valid codomain is $\mathbb{R}$.

Question 10. [1 MARK]

A polynomial of degree 3 or less is a function that looks like $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, where $a_0, a_1, a_2, a_3 \in \mathbb{R}$ are considered to be fixed constants (i.e., they are not variables of the function f). Which of the following is a set of all functions that have domain $\mathbb{R}$ and are polynomials of degree 3 or less?

a. $\{f : \mathbb{R} \to \mathbb{R} \mid f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \text{ and } a_0, a_1, a_2, a_3 \in \mathbb{R}\}$

b. $\{f : \mathbb{R} \to \mathbb{R} \mid f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \text{ and } a_0, a_1, a_2, a_3 \in \mathbb{N}\}$

c. $\{f : \mathbb{R} \to \mathbb{R} \mid f(x) = a_0 + a_1x + a_2x^2 \text{ and } a_0, a_1, a_2 \in \mathbb{R}\}$

d. $\{f : \mathbb{R} \to \mathbb{R} \mid f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 \text{ and } a_0, a_1, a_2, a_3, a_4 \in \mathbb{R}\}$

Question 11. [1 MARK]

Which of the following is a function

$$\mathcal{A} : (\mathbb{R} \times \mathbb{R})^n \to \{f \mid f : \mathbb{R} \to \mathbb{R} \text{ and } f(x) = xw,\ w \in \mathbb{R}\}$$

where $n = 2$?

a. $\mathcal{A}((a, b), (c, d)) = f(x) = ax + b$

b. $\mathcal{A}((a, b), (c, d)) = f(x) = cx$

c. $\mathcal{A}((a, b), (c, d)) = f(x) = ax + d$

d. $\mathcal{A}((a, b), (c, d)) = f(x) = \frac{ab+cd}{a^2+b^2}x$

Question 12. [1 MARK]

Let $f(x, w) = xw$ where $x, w \in \mathbb{R}$. What is the partial derivative of $f(x, w)$ with respect to w?

a. x

b. w

c. $x + w$

d. 1

Question 13. [1 MARK]

Let $\ell(\hat{y}, y) = (\hat{y} - y)^2$ where $\hat{y}, y \in \mathbb{R}$. What is the partial derivative of $\ell(\hat{y}, y)$ with respect to $\hat{y}$?

a. $2(\hat{y} - y)$

b. $(\hat{y} - y)$

c. $2\hat{y}$

d. $-2y$

Question 14. [1 MARK]

Let $\ell(\hat{y}, y) = (\hat{y} - y)^2$ where $\hat{y}, y \in \mathbb{R}$. Let $f(x, w) = xw$ where $x, w \in \mathbb{R}$. What is the partial derivative of $\ell(f(x, w), y)$ with respect to w?

a. $2x(xw - y)$

b. $2(xw - y)$

c. x

d. $2xw$

Question 15. [1 MARK]

If $\mathcal{X} = \{1, 2, 3\}$, what is $\frac{1}{3}\sum_{x \in \mathcal{X}} x$?

Question 16. [1 MARK]

If $\mathcal{X} = (x_1, \ldots, x_n)$, how would you write the sum over all $x \in \mathcal{X}$?

a. $\sum_{i=1}^{n} x_i$

b. $\prod_{x \in \mathcal{X}} x$

c. $\int_{\mathcal{X}} x \, dx$

d. $\sum_{x \in \mathcal{X}} x$

Question 17. [1 MARK]

Let $\mathcal{X} = (x_1, \ldots, x_n)$. Let $f(x, w) = xw$ where $x, w \in \mathbb{R}$. How would you write the sum of $f(x, w)$ over all $x \in \mathcal{X}$?

a. $\sum_{i=1}^{n} x_i w$

b. $w \sum_{i=1}^{n} x_i$

c. $\sum_{x=1}^{n} w$

d. $w \prod_{i=1}^{n} x_i$

Question 18. [1 MARK]

Let $\mathcal{D} = ((x_1, y_1), \ldots, (x_n, y_n))$. Let $\ell(\hat{y}, y) = (\hat{y} - y)^2$ where $\hat{y}, y \in \mathbb{R}$. Let $f(x, w) = xw$ where $x, w \in \mathbb{R}$. How would you write the sum of $\ell(f(x, w), y)$ over all $(x, y) \in \mathcal{D}$?

a. $\sum_{i=1}^{n} (\hat{y}_i - y_i)^2$

b. $\sum_{i=1}^{n} (x_i w - y_i)^2$

c. $\sum_{i=1}^{n} x_i w y_i$

d. $\prod_{i=1}^{n} (x_i w - y_i)^2$

Question 19. [1 MARK]

Let $L(z_1, \ldots, z_n) = \frac{1}{n} \sum_{i=1}^{n} z_i$ where $z_i \in \mathbb{R}$ for all $i \in \{1, \ldots, n\}$. Let $\ell(\hat{y}, y) = (\hat{y} - y)^2$ where $\hat{y}, y \in \mathbb{R}$. Let $f(x, w) = xw$ where $x, w \in \mathbb{R}$. What is the partial derivative of $L(\ell(f(x_1, w), y_1), \ldots, \ell(f(x_n, w), y_n))$ with respect to w?

a. $\frac{2}{n} \sum_{i=1}^{n} x_i (x_i w - y_i)$

b. $\sum_{i=1}^{n} x_i (x_i w - y_i)$

c. $\frac{1}{n} \sum_{i=1}^{n} 2(x_i w - y_i)$

d. $2 \sum_{i=1}^{n} x_i w$

Question 20. [1 MARK]

Let $f(x, y) = xy$ where $x, y \in \mathbb{R}$. What is

$$\sum_{y \in \mathcal{Y}} f(x, y)$$

where $\mathcal{Y} = \{1, 2, 3\}$?

a. $6x$

b. 6

c. $6y$

d. x

Question 21. [1 MARK]

Let $f(x, y) = xy$ where $x, y \in \mathbb{R}$. What is

$$\int_{\mathcal{X}} \sum_{y \in \mathcal{Y}} f(x, y)\, dx = \int_{\mathcal{X}} \left(\sum_{y \in \mathcal{Y}} f(x, y) \right) dx$$

where $\mathcal{Y} = \{1, 2, 3\}$ and $\mathcal{X} = [0, 5]$?

Question 22. [1 MARK]

Let $f(x, y) = xy$ where $x, y \in \mathbb{R}$. What is

$$\int_{\mathcal{Y}} f(x, y)\, dy$$

where $\mathcal{Y} = [1, 3]$?

a. $4x$

b. $6x$

c. $2x$

d. x

Question 23. [1 MARK]

Let $f(x, y) = xy$ where $x, y \in \mathbb{R}$. What is

$$\int_{\mathcal{X}} \int_{\mathcal{Y}} f(x, y)\, dy\, dx = \int_{\mathcal{X}} \left(\int_{\mathcal{Y}} f(x, y)\, dy \right) dx$$

where $\mathcal{Y} = [1, 3]$ and $\mathcal{X} = [0, 5]$?

Question 24. [1 MARK]

Let X be a discrete random variable uniformly distributed with outcome space $\mathcal{X} = \{3, 5, 7, 9\}$. The probability mass function (pmf) of X is given by $p(x) = \frac{1}{4}$ for each $x \in \mathcal{X}$. What is the probability of the event that X is either 5 or 9?

Question 25. [1 MARK]

Let Y be a continuous random variable uniformly distributed with outcome space $\mathcal{Y} = [2, 10]$. The probability density function (pdf) of Y is given by $p(y) = \frac{1}{8}$ for $y \in [2, 10]$. What is the probability of the event that Y lies between 4 and 7?